Introduction to Matrices

1 VECTOR EQUATIONS, MATRIX EQUATIONS, AND LINEAR SYSTEMS

Convert the equations below to systems of linear equations and solve for x and y.

(a) $\begin{bmatrix}1\\2\end{bmatrix}x+\begin{bmatrix}2\\5\end{bmatrix}y=\begin{bmatrix}0\\1\end{bmatrix}$ (b) $\begin{bmatrix}3\\2\end{bmatrix}x+\begin{bmatrix}2\\4\end{bmatrix}y=\begin{bmatrix}-1\\-6\end{bmatrix}$ (c) $\begin{bmatrix}2&1\\-2&3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}5\\-1\end{bmatrix}$ (d) $\begin{bmatrix}2&-1\\2&3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\8\end{bmatrix}$

Convert the matrix equations below to a vector equation and system of equations. Graph the lines represented by the system of equations to find their intersection. Draw the vectors given by the vector equation to check its solution.

(e) $\begin{bmatrix}1&1\\1&-1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}5\\1\end{bmatrix}$ (f) $\begin{bmatrix}1&2\\2&0\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}0\\4\end{bmatrix}$

2 MULTIPLICATION

Multiply the following matrices. (All products below are defined.)

(a) $\begin{bmatrix}1&2\\3&4\end{bmatrix}\begin{bmatrix}2\\-1\end{bmatrix}$ (b) $\begin{bmatrix}2&-1\end{bmatrix}\begin{bmatrix}1&2\\3&4\end{bmatrix}$ (c) $\begin{bmatrix}1&0\\2&1\end{bmatrix}\begin{bmatrix}1&2\\3&4\end{bmatrix}$ (d) $\begin{bmatrix}1&2\\3&4\end{bmatrix}\begin{bmatrix}1&0\\2&1\end{bmatrix}$

(e) $\begin{bmatrix}2&1\\0&1\\1&0\end{bmatrix}\begin{bmatrix}1&-1\\2&1\end{bmatrix}$ (f) $\begin{bmatrix}1&0&1\end{bmatrix}\begin{bmatrix}2\\1\\-3\end{bmatrix}$ (g) $\begin{bmatrix}1&0&1\\0&1&1\end{bmatrix}\begin{bmatrix}2\\1\\-3\end{bmatrix}$ (h) $\begin{bmatrix}2&0\\0&4\\0&0\end{bmatrix}\begin{bmatrix}2&1&-3\\0&2&1\end{bmatrix}$

3 DIVISION WITH TRIANGULAR MATRICES

Use forward or backward substitution to divide. (Problems *(e)*-*(h)* are more advanced!)
Some problems may have no solutions, or infinitely many solutions.

(a) $\begin{bmatrix}1&0&0\\2&1&0\\3&2&1\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\4\\4\end{bmatrix}$ (b) $\begin{bmatrix}2&1&3\\0&1&2\\0&0&3\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\1\\6\end{bmatrix}$

(c) $\begin{bmatrix}1&2&3\\0&0&2\\0&0&0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\6\\0\end{bmatrix}$ (d) $\begin{bmatrix}1&2&3\\0&0&2\\0&0&0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\6\\1\end{bmatrix}$

(e) $\begin{bmatrix}x&y&z\end{bmatrix}\begin{bmatrix}1&2&3\\0&2&2\\0&0&1\end{bmatrix}=\begin{bmatrix}3&4&4\end{bmatrix}$ (f) $\begin{bmatrix}x&y&z\end{bmatrix}\begin{bmatrix}1&0&0\\2&2&0\\3&1&1\end{bmatrix}=\begin{bmatrix}3&4&4\end{bmatrix}$

(g) $\begin{bmatrix}a&b&c\\d&e&f\\g&h&i\end{bmatrix}\begin{bmatrix}1&2&3\\0&2&2\\0&0&1\end{bmatrix}=\begin{bmatrix}1&2&3\\1&4&5\\0&0&1\end{bmatrix}$ (h) $\begin{bmatrix}1&2&3\\0&2&2\\0&0&1\end{bmatrix}\begin{bmatrix}a&b&c\\d&e&f\\g&h&i\end{bmatrix}=\begin{bmatrix}1&3&3\\0&2&2\\0&0&1\end{bmatrix}$

4 MatLab

- Matrices are entered into MatLab using square brackets. Commas or spaces separate numbers on the same row, semicolons or new lines separate rows. For example the commands

```
>> A = [1, 2, 3, 4 ; 5, 6, 7, 8]
>> A = [1 2 3 4 ; 5 6 7 8]
>> A = [1 2 3 4
        5 6 7 8]
```

  all create the same matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix}$

- Matrix multiplication is indicated with the $*$ symbol. For example the command

```
>> [1 2 3 ; 5 6 7] * [1 -1 ; 3 2 ; 5 7]
```

  will compute the matrix product $\begin{bmatrix} 1 & 2 & 3 \\ 5 & 6 & 7 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 3 & 2 \\ 5 & 7 \end{bmatrix}$

- Recall that it is not always possible to multiply matrices. For example, $\begin{bmatrix} 1 & 3 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & 1 \\ 1 & 1 \end{bmatrix}$ is not defined because the right matrix has 3 rows (3 outputs) but the left matrix has only 2 columns (2 inputs). MatLab will throw an error if you ask it to perform this multiplication.

```
>> [1 3 ; 4 -1] * [0 1 ; 2 1 ; 1 1]
 Error using  *
 Inner matrix dimensions must agree.
```

  This is the most common error to see when using MatLab. Get used to it.

- Matrix division is indicated with the / and \ symbols for division on the left and right.

  For example, to solve the division problem $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$ you divide on the left

```
>> [1 2; 3 4] \ [5; 7]

 ans =
     -3
      4
```

  To solve the division problem $\begin{bmatrix} x & y \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5 & 7 \end{bmatrix}$ you divide on the right

```
>> [5 7] / [1 2; 3 4]

 ans =
    0.5000    1.5000
```

  Note that x and y are different in these two examples!

Go back to sections 2 and 3 on the first page and solve the problems again, this time using MatLab!